Hong Kong Mathematics Olympiad (2004 – 2005)
Final Event 1 (Individual)
香港数学竞赛 (2004 – 2005)
决赛项目 1 (个人)

除非特别声明，答案须用数字表达，并化至最简。
Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若在 1 至 200 内能同时被 3 和 7 整除的数有 a 个，求 a 的值。

Suppose there are a numbers between 1 and 200 that can be divisible by 3 and 7, find the value of a.

2. 设质数 p 和 q 是方程 $x^2-13x+R=0$ 的两个不同的根，其中 R 是实数。若 $b=p^2+q^2$，求 b 的值。

Let p and q be prime numbers that are the two distinct roots of the equation $x^2-13x+R=0$, where R is a real number. If $b=p^2+q^2$, find the value of b.

3. 已知 $\tan\alpha=-\frac{1}{2}$。若 $c=\frac{2\cos\alpha-\sin\alpha}{\sin\alpha+\cos\alpha}$，求 c 的值。

Given that $\tan\alpha=-\frac{1}{2}$. If $c=\frac{2\cos\alpha-\sin\alpha}{\sin\alpha+\cos\alpha}$, find the value of c.

4. 设 r 和 s 是方程 $2\left(x^2+\frac{1}{x^2}\right)-3\left(x+\frac{1}{x}\right)=1$ 的两个不同的实数根。若 $d=r+s$，求 d 的值。

Let r and s be the two distinct real roots of the equation $2\left(x^2+\frac{1}{x^2}\right)-3\left(x+\frac{1}{x}\right)=1$. If $d=r+s$, find the value of d.

Hong Kong Mathematics Olympiad (2004 – 2005)
Final Event 2 (Individual)
香港数学竞赛 (2004 – 2005)
决赛项目 2 (个人)

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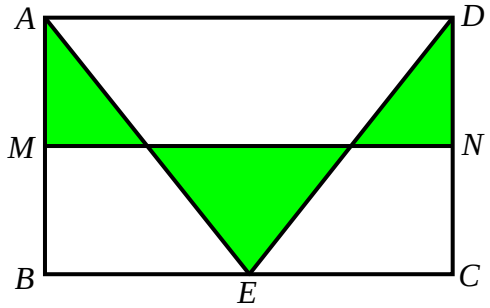
1.



图一
Figure 1

如图一，在长方形 $ABCD$ 中， $AB = 6$ cm，$BC = 10$ cm。M 和 N 分别是 AB 和 DC 的中点。若阴影部分的面积是 a cm^2，求 a 的值。

In Figure 1, $ABCD$ is a rectangle, $AB = 6$ cm and $BC = 10$ cm . M and N are the midpoints of AB and DC respectively. If the area of the shaded region is a cm^2 , find the value of a .

2. 设 $b = 89 + 899 + 8999 + 89999 + 899999$，求 b 的值。

Let $b = 89 + 899 + 8999 + 89999 + 899999$, find the value of b .

3. 已知 $2x + 5y = 3$。若 $c = \sqrt{4^{x+\frac{1}{2}} \times 32^y}$，求 c 的值。

Given that $2x + 5y = 3$. If $c = \sqrt{4^{x+\frac{1}{2}} \times 32^y}$, find the value of c .

4. 设 $d=\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+\ldots+\frac{10}{2^{10}}$，求 d 的值。

Let $d=\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+\ldots+\frac{10}{2^{10}}$, find the value of d .

Hong Kong Mathematics Olympiad (2004 – 2005)
Final Event 3 (Individual)
香港数学竞赛 (2004 – 2005)
决赛项目 3 (个人)

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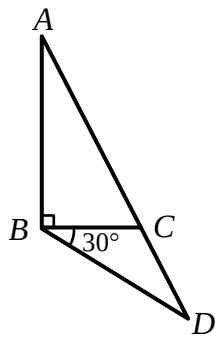
1. 设 $0° < \alpha < 45°$。若 $\sin\alpha\cos\alpha = \frac{3\sqrt{7}}{16}$ 及 $A = \sin\alpha$，求 A 的值。

Let $0° < \alpha < 45°$. If $\sin\alpha\cos\alpha = \frac{3\sqrt{7}}{16}$ and $A = \sin\alpha$, find the value of A.

2.

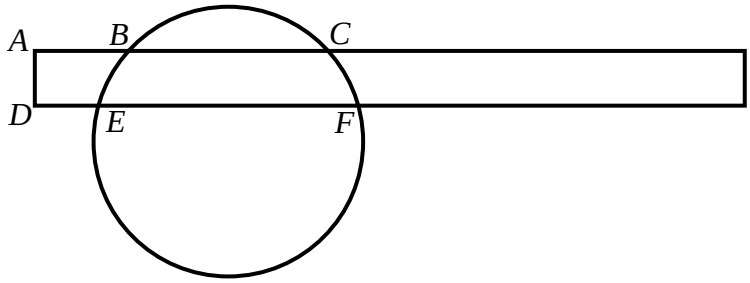


图一
Figure 1

如图一，C 在 AD 上且 $AB = BD = 1\text{cm}$，$\angle ABC = 90°$，$\angle CBD = 30°$。若 $CD = b\text{cm}$，求 b 的值。

In figure 1, C lies on AD, $AB = BD = 1\text{cm}$, $\angle ABC = 90°$ and $\angle CBD = 30°$. If $CD = b\text{cm}$, find the value of b.

3.



图二

Figure 2

如图二，一长方形与圆相交于点 B 、 C 、 E 及 F。已知 $AB = 4$ cm， $BC = 5$ cm 及 $DE = 3$ cm。若 $EF = c$ cm，求 c 的值。

In Figure 2, a rectangle intersects a circle at points B, C, E and F. Given that $AB = 4$ cm, $BC = 5$ cm and $DE = 3$ cm. If $EF = c$ cm, find the value of c.

4. 假设 x 和 y 都是正数并且成反比。若 x 增加了 10 %，则 y 减少了 d %，求 d 的值。

Let x and y be two positive numbers that are inversely proportional to each other. If x is increased by 10 %, y will be decreased by d %, find the value of d.

Hong Kong Mathematics Olympiad (2004 – 2005)
Final Event 4 (Individual)
香港数学竞赛 (2004 – 2005)
决赛项目 4 (个人)

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1. 若 $a=\log_{\frac{1}{2}}0.125$，求 a 的值。

If $a=\log_{\frac{1}{2}}0.125$, find the value of a.

2. 若方程 $\left|x-|2x+1|\right|=3$ 有 b 个不同的解，求 b 的值。

Suppose there are b distinct solutions of the equation $\left|x-|2x+1|\right|=3$, find the value of b.

3. 若 $c=2\sqrt{3}\times\sqrt[3]{1.5}\times\sqrt[6]{12}$，求 c 的值。

If $c=2\sqrt{3}\times\sqrt[3]{1.5}\times\sqrt[6]{12}$, find the value of c.

4. 已知 $f_1=0$，$f_2=1$ 及对正整数 $n\geq 3$，$f_n=f_{n-1}+2f_{n-2}$。若 $d=f_{10}$，求 d 的值。

Given that $f_1=0$, $f_2=1$ and for any positive integer $n\geq 3$, $f_n=f_{n-1}+2f_{n-2}$. If $d=f_{10}$, find the value of d.